# On the Rheological Behavior of Frozen Soil (Part I)



FUKUO, Yoshiaki

Bulletin of the Disaster Prevention Research Institute, Vol. 15, Part 3, No. 96, March, 1966

# On the Rheological Behavior of Frozen Soil (Part I)

By Yoshiaki FUKUO

(Manuscript received January 13, 1966)

**Abstract**

In order to examine the rheological behavior of frozen soil, an experiment of compression in axial direction was carried out, using the samples moulded into cylindrical form with 5 cm in diam. and 9 cm in height. The results of the experiment were analysed according to the Murayama and Shibata theory for the rheological characters of clay, and it may be considered that the upper yield value of frozen silty clay was between 10 and 20 kg/cm$^2$ at $-5$°C and greater than 20 kg/cm$^2$ at $-10$°C and also its lower yield value was about 2~4 kg/cm$^2$. The deformation of frozen silty sand was greater than that of silty clay under the same temperature and load in all samples examined here.

## 1. Introduction

It is well knowm that the water in soil influences significantly its physical and chemical properties. When the soil is cooled and water in it is frozen, the interesting phenomena occur such as the heaving of soil, the remaining of unfrozen water and the increasing of mechanical strength of soil, which play the important roles in the development of niveal geomorphic features[1]. These phenomena are not yet fully understood today.

Recently in our country, utilizing the increase of strength practically, freezing method of soil in engineering construction is in progress. For the safety of construction, the rheological behavior of frozen soil must be known exactly and fully. So, an experiment was carried out to examine its behavior.

## 2. Soil samples

Since May 1965, earth soil had been frozen near Kanasugi Bridge in Tokyo for the preexamination of the freezing method of construction, and a vertical pit with diam. 2 m was bored there from surface to the depth of 10 m. On the sidewall of this pit, the blocks of frozen soil were sampled at 5, 7, 8.5 and 10 m depth ; two blocks to each depth, the one for undisturbed and the other for remoulded soil. All blocks were packed doubly by vinyl sacks to prevent the evaporation of water in soil.

Blocks for undisturbed sample were preserved in a freezer kept at a temperature of $-25$°C$\pm$0.5°C about three months and being scrapped off by trimmer, they were formed into a cylinder with 5 cm in diam. and 9 cm in height. Soon after a test piece was trimmed, it was coated by grease all over its surface and was set on the base of axial compressive device which was put in an ice box. Blocks for remoulded sample were melted and stuffed into the hole of moulding case with the same cylindrical size as stated above. The case was immerged two days in a brine box kept at a temperature of $-23°C$ $\pm 1°C$ and stuffed soil was frozen again. Soon after a test piece was remoulded it was set on the base in the same way for undisturbed sample.

The blocks at 5, 7 and 8.5 m depth were silty clays consolidated considerably. The blocks at 10 m depth were silty sands containing gravels and debris of shells. For the silty clays the consistency limits and water content and for the silty sand the grain size distribution and water content were measured from the scrapps of undisturbed blocks. The results are tabulated in Tables 1 and 2.

TABLE 1.
Consistency limits and water contents of silty clay scrapped from undisturbed frozen blocks sampled at 5, 7 and 8.5 m depth layers.

| depth (m) | 5 | 7 | 8.5 |
|---|---|---|---|
| plastic limit (%) | 37.0 | 37.6 | 35.4 |
| liquid limit (%) | 71.0 | 70.2 | 70.7 |
| water content (%) | 51.0 | 52.7 | 64.6 |

TABLE 2.
Grain size distribution and water content of silty sand scrapped from undisturbed frozen block sampled at 10 m depth layer.

| | | |
|---|---|---|
| | analysed total weigh in dry | 390.9 gr |
| | water content | 25.0 % |
| mesh size ($\mu$) | >2380 | 2380> |
| weigt (gr) | 234.2 | 155.8 |
| | sieving loss | 0.9 gr |

Grain size distribution less than 2380 $\mu$ mesh

| mesh size ($\mu$) | >2000 | >1410 | >1000 | >710 | >500 | >350 | >250 | >210 | 210> |
|---|---|---|---|---|---|---|---|---|---|
| weight percentage | 11.4 | 18.6 | 15.5 | 14.0 | 11.0 | 17.4 | 5.6 | 1.6 | 4.9 |

## 3. Instrument and method of experiment

The schema of axial compressive device was drawn in Fig. 1. The weight (W) was hung at point (P). The test piece was compressed in its axial direction by the weight (W) multiplied by twenty fold through the lever arms (A) and (B), shaft (C) and plate (D). Photo 1 shows the view of device.

The deformations of piece in axial and radial (I), (II) directions were converted to the changes of D. C. voltage by the linear differential transformers and detected on the dotting auto-recorder with accuracy of $\pm 0.04$ mm.

The compressive device was placed in an ice box which had a net capacity of 0.7 m in width, 0.8 m in depth and 0.9 m in height and was cooled by an electric refrigerator. The temperature in the box was controlled at constant temperature by a thermostat and stirring fan with accuracy of $\pm 0.5°C$.

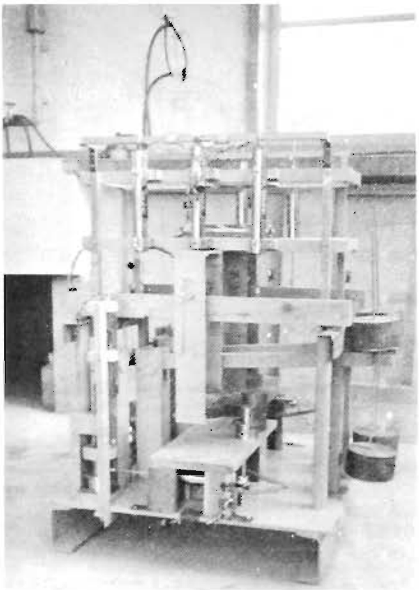

Photo. 1. The view of axial compressive device.

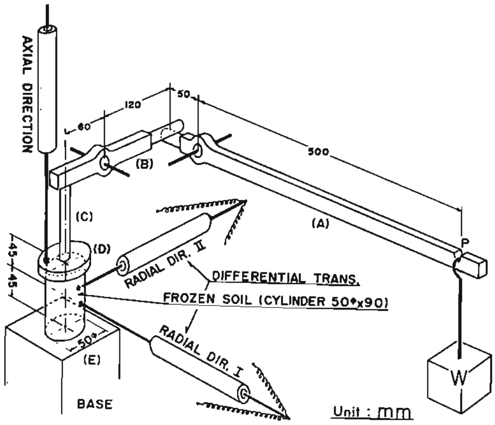


Fig. 1. The schema of axial compressive device.

Loading and unloading of weights forced the opening of the door of the ice box for about two minutes and consequetly disturbed its temperature by the inflow of outer air. So, dotted records for about five minutes after loading and unloading did not show the deformations of piece at constant temperature and weights.

## 4. Result of experiment and its consideration

The deformations of undisturbed pieces at 8.5 and 10 m layers are shown in Fig. 2 as typical examples. In this figure, it was seen apparently that the deformations represented creeping phenomena in both examples and the creep in silty clay was smaller than that in silty sand under the same constant load and temperature. In each example, the test pieces were expanded in radial direction (I), as expected naturally, according to the contraction in axial direction, but the deformations in direction (II) was contrary to those in direction (I). Such a tendency was found in almost all the results of the experiment. It may be caused by the inconsistency between the directions of compressive force and cylindrical axis of the piece. Radial deformation will be analysed in future papers.

Now, the rheological properties of unfrozen clay was theoretically illustrated by Dr. Murayama and Shibata[2]. The freezing effect of water in clay and its behavior are not yet known fully and remains to be solved in the future. So, it will be valuable to examine whether the rheological behavior of frozen soil is similar to that of unfrozen soil or not. According to their theory, the contractive strain $\varepsilon_c$ of clay in axial direction is given by

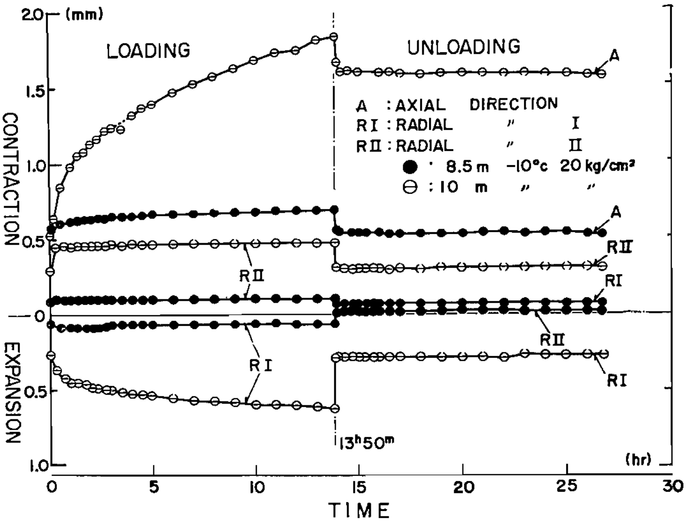


Fig. 2. Typical examples of deformations of undisturbed pieces.

$$\varepsilon_c=\frac{\sigma}{E_1}+\frac{\sigma-\sigma_0}{E_2}-\frac{2(\sigma-\sigma_0)}{B_2E_2}\tanh^{-1}\left[\exp\left(-A_2B_2E_2t_c\right)\tanh\frac{B_2}{2}\right] \qquad (1),$$

under constant pressure $\sigma$ smaller than the upper yield value $\sigma_u$, where

$E_1$ : Young's modulus between the matrixes of clay particles,
$E_2$ : Young's modulus in a matrix,
$\sigma_0$ : lower yield value of clay,
$t_c$ : time measured from loading,
$A_2$ and $B_2$ : proper constants of clay depending on temperature.

When

$$\frac{B_2}{2}>1 \text{ and } \frac{\sigma}{E_1}<\varepsilon_c<\frac{\sigma}{E_1}+\frac{\sigma-\sigma_0}{2B_2E_2}(2B_2-1) \qquad (2),$$

Eq. (1) is represented approximately by

$$\varepsilon_c=\frac{\sigma}{E_1}+\frac{\sigma-\sigma_0}{E_2}+\frac{\sigma-\sigma_0}{B_2E_2}\log_e\left(\frac{1}{2}A_2B_2E_2t_c\right) \qquad (3).$$

Eq. (3) indicates that the strain $\varepsilon_c$ is related linearly with the logarithm of time $t_c$ in case where $\sigma<\sigma_u$. When $\sigma>\sigma_u$, the value of $\varepsilon_c$ is increased concavely upward with $\log_e t_c$ on the semi-logarithmic graph.

After unloading at $t_c=t_1$, elastic strain $\frac{\sigma}{E_1}$ is recovered instantaneously and the strain $\varepsilon_r$ in recovery stage is given by

$$\varepsilon_r=\frac{\sigma_0}{E_2}+\frac{2\sigma_0}{B_2E_2}\tanh^{-1}\left[\exp\left(-A_2B_2E_2t_r\right)\tanh\frac{B_2}{2\sigma_0}(\varepsilon_aE_2-\sigma_0)\right] \qquad (4),$$

where time $t_r$ is taken from unloading and the strain $\varepsilon_a$ is the value of $\varepsilon_c$ minus $\frac{\sigma}{E_1}$ at $t_c=t_1$, that is, $t_r=0$.

When

$$\frac{B_2}{2} > 1 \text{ and } \varepsilon_a > \varepsilon_r > \frac{\sigma_0}{2B_2E_2}(2B_2+1) \tag{5},$$

Eq. (4) is represented approximately by

$$\varepsilon_r = \frac{\sigma_0}{E_2} - \frac{\sigma_0}{B_2E_2}\log_e\left(\frac{1}{2}A_2B_2E_2t_r\right) \tag{6}.$$

Eq. (6) indicates that the strain $\varepsilon_r$ is related linearly with $\log_e t_r$ in any recovery stage.

Refering to the above theory, the results of the experiment were analysed on semi-logarithmic graphs as shown in Figs. 3, 4 and 5, where the figures in parentheses and circles represented constant loads and reference numbers of experiments respectively. In Fig. 3, the contractions of 7 and 8.5 m pieces were arranged on straight lines in case where $\sigma = 10\,\text{kg/cm}^2$ and nearly equal except for the line No. 3. This line represented the second creep of the piece which was experienced in the first creep shown by curve No. 4. The latter was concave upward under $\sigma = 20\,\text{kg/cm}^2$. In Fig. 4, undisturbed pieces of 7 and 8.5 m layers showed quite the same straight lines of creep under $\sigma = 20\,\text{kg/cm}^2$ and remoulded pieces of same layers showed also the straight lines, although smaller deformations than those of undisturbed pieces. It may be infered that undisturbed frozen clays of 7 and 8.5 m layers had the upper yield value $\sigma_u$ between 10 and $20\,\text{kg/cm}^2$ at $-5°\text{C}$ and above $20\,\text{kg/cm}^2$ at $-10°\text{C}$.

The contractions of silty sand at 10 m layer was great compared with those of silty clay under the same load in all cases examined here, and seemed to

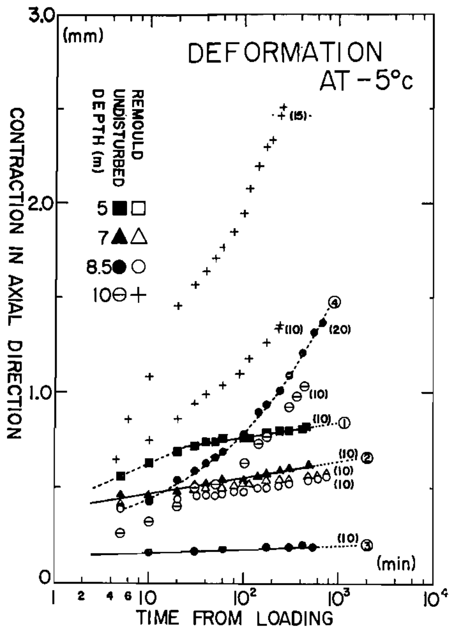


Fig. 3. Deformations pieces during loading at $-5°\text{C}$.

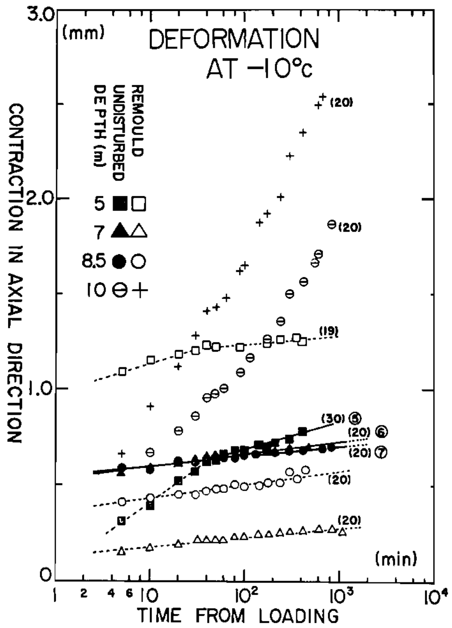


Fig. 4. Deformations of pieces during loading at $-10°\text{C}$.

increase concavely upward with the logarithm of time. These results may suggest the existence of difference of rheological character between frozen sand and clay.

From the coefficient of $\log_e t$ in Eqs. (3) and (6), the lower yield value $\sigma_0$ can be calculated as follows. Noting that

$$b_1 \equiv \frac{\sigma - \sigma_0}{B_2 E_2} \text{ and } b_2 \equiv \frac{\sigma_0}{B_2 E_2} \quad (7),$$

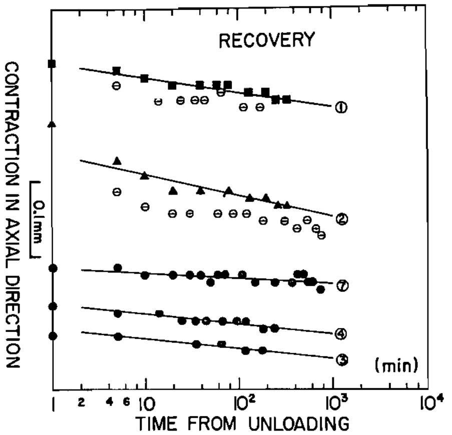

Fig. 5. Recovery of deformations.

then

$$\sigma_0 = \frac{b_2}{b_1 + b_2}\sigma \quad (8).$$

Fig. 5 shows the relation between the strain and logarithm of time on recovery stage. Recoveries for No. 5 and 6 were not drawn because the change of strain could not be found with the accuracy of measurement after the elastic recovery. Using the results of measurement, the values of $b_1$ and $b_2$ were calculated by least squares method and were tabulated in Table 3 with constant terms $a_1$ and $a_2$. At $-5°C$

TABLE 3.

Results of calculations by least squares method for the straight lines shown in Figs. 3, 4 and 5.

−5°C

| depth (m) | reference number | compressive load $\sigma$ (kg/cm²) | loading | | unloading | | $\sigma_0$ (kg/cm²) $\left(=\frac{b_2\sigma}{b_1+b_2}\right)$ |
|---|---|---|---|---|---|---|---|
| | | | $a_1 \times 10^3$ | $b_1 \log_e 10 \times 10^4$ | $a_2 \times 10^2$ | $b_2 \log_e 10 \times 10^4$ | |
| 5 | ① | 10 | 6.8 | 8.7 | 8.1 | 2.2 | 2.0 |
| 7 | ② | 10 | 4.2 | 9.8 | 5.7 | 3.1 | 2.4 |
| 8.5 | ③ | 10 | 1.6 | 2.2 | 0.9 | 1.4 | 3.9 |

−10°C

| depth (m) | reference number | compressive load $\sigma$ (kg/cm²) | loading | | unloading | | $\sigma_0$ (kg/cm²) $\left(=\frac{b_2\sigma}{b_1+b_2}\right)$ |
|---|---|---|---|---|---|---|---|
| | | | $a_1 \times 10^3$ | $b_1 \log_e 10 \times 10^4$ | $a_2 \times 10^3$ | $b_2 \log_e 10 \times 10^4$ | |
| 5 | ⑤ | 30 | 4.6 | 1.5 | 5.5 | — | — |
| 7 | ⑥ | 20 | 5.9 | 7.3 | 6.7 | — | — |
| 8.5 | ⑦ | 20 | 6.0 | 5.9 | 6.2 | 0.8 | 2.3 |

$$\text{where } a_1 \equiv \frac{\sigma}{E_1} + \frac{\sigma - \sigma_0}{E_2} + \frac{\sigma - \sigma}{B_2 E_2} \log_e\left(\frac{1}{2} A_2 B_2 E_2\right)$$

$$a_2 \equiv \frac{\sigma}{E_1} + \frac{\sigma_0}{E_2} - \frac{\sigma_0}{B_2 E_2} \log_e\left(\frac{1}{2} A_2 B_2 E_2\right)$$

and $-10°C$, the obtained values of $\sigma_0$ were between 2 and 4 kg/cm$^2$ for frozen clay.

## 5. Conclusion

An experiment of compression in axial direction was carried out, using frozen soils sampled at an alluvial plane in Tokyo and from the results of the experiment it may be considered that

i) the upper yield value of frozen silty clay was between 10 and 20 kg/cm$^2$ at $-5°C$ and greater than 20 kg/cm$^2$ at $-10°C$

ii) the lower yield value of frozen silty clay was about 2~4 kg/cm$^2$.

iii) the deformation of frozen silty sand was greater than that of silty clay under the same temperature and load.


### Acknowledgements

The writer wishes to experess his sincere appreciation to Professors S. Okuda and S. Murayama of Kyoto University for their cordial guidance and encouragement in this work.

His thanks are also due to Dr. T. Takashi and T. Kato, members of staff of the Seiken-Reiki Co., Ltd. for their hearty support.


### References


1) Scheidegger, A. D., "Theoretical Geomorphology", Springer-Verlag, Berlin, 1961, pp. 281–286.

2) Murayama, S. and Shibata, T., On the Rheological Characters of Clay, Trans. of JSCE, No. 40, pp. 1–31.